The Problem Corner

Ivan Retamoso, PhD, *The Problem Corner* Editor

Borough of Manhattan Community College

iretamoso@bmcc.cuny.edu

The Purpose of **The Problem Corner** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Hello Problem Solvers, I got solutions to **Problem 8** and to **Problem 9** and I am happy to inform that they were correct, interesting, and ingenious. By posting what I considered to be the best solutions, I hope to enrich and enhance the mathematical knowledge of our international community.

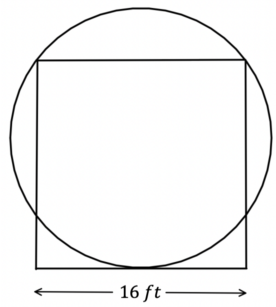
Solutions to **Problems** from the Previous Issue

Interesting Geometry problem.

Proposed by Ivan Retamoso from Borough of Manhattan Community College, City university of New York, USA

Problem 8

In the figure below find the ratio between the perimeter of the circle (circumference) and the perimeter of the square in exact form.

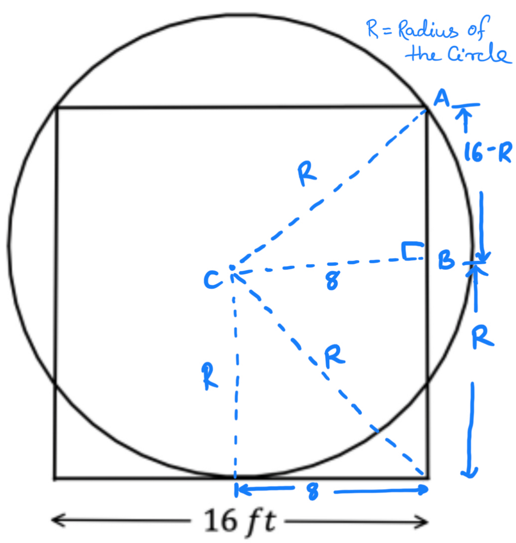


Solution to problem 8

Aradhana Kumari, Borough of Manhattan Community College, City university of New York, USA.

This clearly explained solution utilizes the theorem of Pythagoras as a tool to find the radius of the circle, which in turn serves to compute the requested ratio.

Consider the diagram below.



Let C be the center of the circle.

Step1. We want to first find the radius of the circle.

Applying Pythagorean theorem to right triangle CBA, we have

(Length of CB)^2 + (length of BA)^2 = (length of CA)^2

$(16\text{-R})^2 + 8^2 = R^2$

$256 + R^2 – 32\ R + 64 = R^2$

Hence

$320 = 32\ R$

$R = \frac{320}{32} = 10$

Hence the circumference of the given circle $= 2\ \pi\ \times R = 2\pi \times 10 = 20\ \pi$

The perimeter of the given square is $4 \times$ length of the square $= 4 \times\ 16 = 64$

Therefore

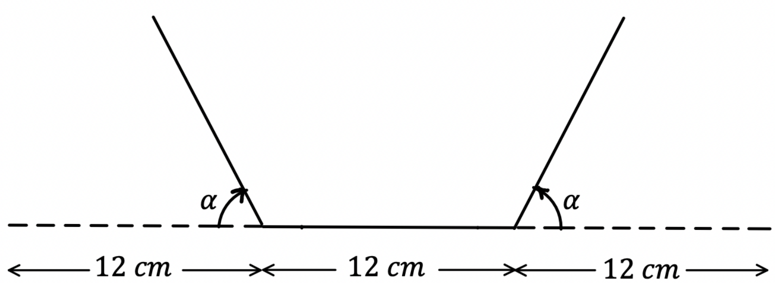
$\frac{\text{circumference of the given circle}}{\text{perimeter of the given square}} = \frac{20\ \pi}{64} = \frac{5\pi}{16}$

Interesting Applied Optimization problem.

Problem 9

Proposed by Ivan Retamoso, BMCC, USA.

It is needed to construct a rain gutter from a metal sheet of width 36 cm by bending up one-third of the sheet on each side through an angle α. Find the value of α such that the gutter will carry the maximum amount of water.

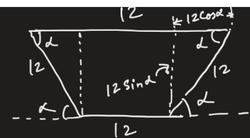


Solution to problem 9

by Aradhana Kumari, Borough of Manhattan Community College, City university of New York, USA.

Sometimes the solution to a problem is independent of some measurements, when we attempt to solve the problem we feel that information is missing, other times we discover that some given quantities are not relevant for the final solution of a problem, often we read, as part of the solution "without loss of generality, let a measure be this or that..", problems like this, in my opinion, are important because the solution can be generalized to an arbitrary context. The solution presented below is a great example of how to solve this type of problems, additionally the solver provided a picture that help to visualize the construction of the gutter with the "perfect" angle for the rain.

Let L be the length of the Container
then Volume of the Container



$$V = L \times \left[12\times 12\sin\alpha + \tfrac{1}{2}\times 12\cos\alpha\times 12\sin\alpha + \tfrac{1}{2}\times 12\cos\alpha\times 12\sin\alpha\right]$$

$$= L\times[144\sin\alpha + 144\sin\alpha\cos\alpha] \quad ---- \quad (1)$$

for volume to be Maximum we differentiate V and equate it to zero.

$$\therefore \frac{dV}{d\alpha} = L\times 144\cos\alpha + L\times 144(\cos\alpha\cos\alpha - \sin\alpha\sin\alpha)$$

$$= 144L\cos\alpha + 144L(\cos^2\alpha - \sin^2\alpha) \quad ---- \quad (2)$$

for Maximum volume

$$\frac{dV}{d\alpha} = 0 \Rightarrow 144L\cos\alpha + 144L(\cos^2\alpha - \sin^2\alpha) = 0$$

$$\Rightarrow 144L[\cos\alpha + \cos^2\alpha - \sin^2\alpha] = 0$$

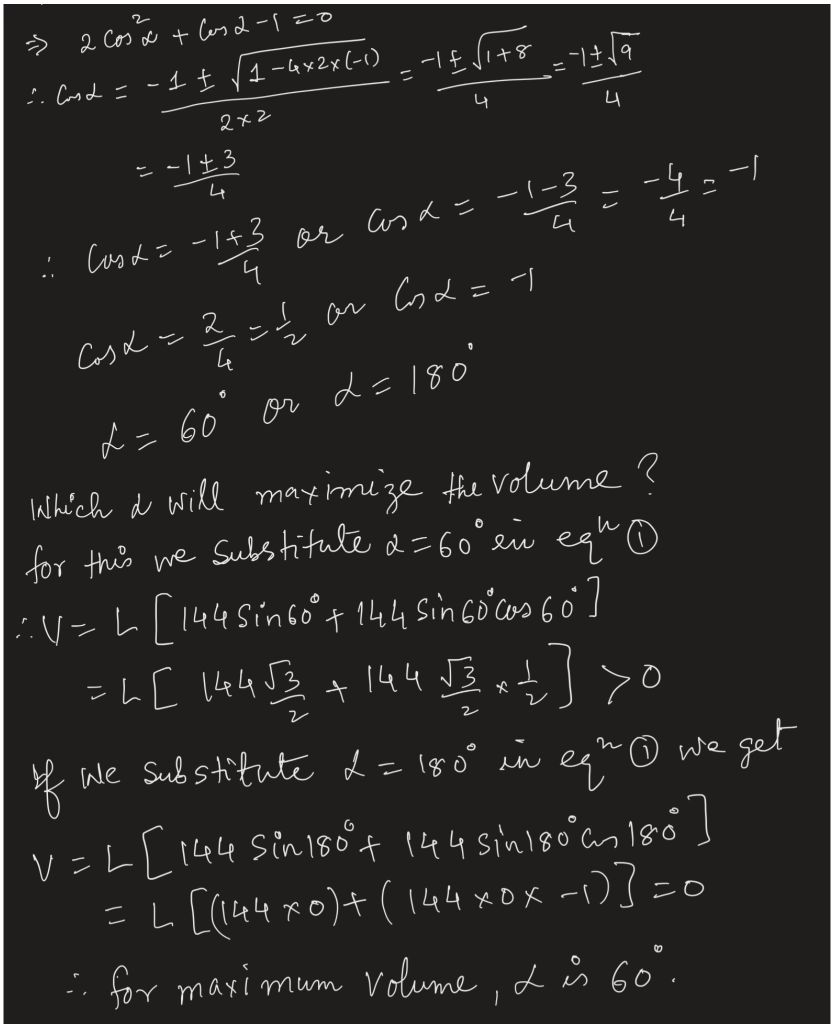
$$\Rightarrow \cos\alpha + \cos^2\alpha - \sin^2\alpha = 0$$

$$\Rightarrow \cos\alpha + \cos^2\alpha - (1 - \cos^2\alpha) = 0$$

$$\Rightarrow \cos\alpha + \cos^2\alpha - 1 + \cos^2\alpha = 0$$

$$\Rightarrow 2\cos^2\alpha + \cos\alpha - 1 = 0$$

$$\therefore \cos\alpha = \frac{-1 \pm \sqrt{1 - 4\times 2\times(-1)}}{2\times 2} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm\sqrt{9}}{4}$$



$\Rightarrow 2\cos^2\alpha + \cos\alpha - 1 = 0$

$\therefore \cos\alpha = \frac{-1 \pm \sqrt{1-4\times2\times(-1)}}{2\times2} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm\sqrt{9}}{4}$

$= \frac{-1\pm3}{4}$

$\therefore \cos\alpha = \frac{-1+3}{4}$ or $\cos\alpha = \frac{-1-3}{4} = \frac{-4}{4} = -1$

$\cos\alpha = \frac{2}{4} = \frac{1}{2}$ or $\cos\alpha = -1$

$\alpha = 60^\circ$ or $\alpha = 180^\circ$

Which α will maximize the volume?

for this we substitute $\alpha = 60^\circ$ in eqn ①

$\therefore V = L[144\sin60^\circ + 144\sin60^\circ\cos60^\circ]$

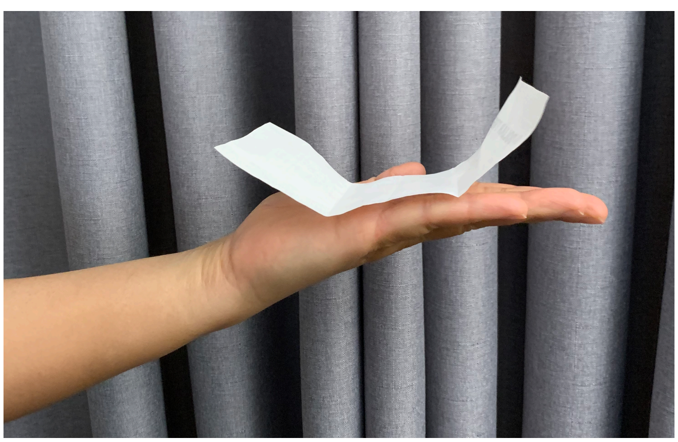
$= L[144\frac{\sqrt3}{2} + 144\frac{\sqrt3}{2}\times\frac{1}{2}] > 0$

If we substitute $\alpha = 180^\circ$ in eqn ① we get

$V = L[144\sin180^\circ + 144\sin180^\circ\cos180^\circ]$

$= L[(144\times0)+(144\times0\times-1)] = 0$

$\therefore$ for maximum volume, α is 60°.



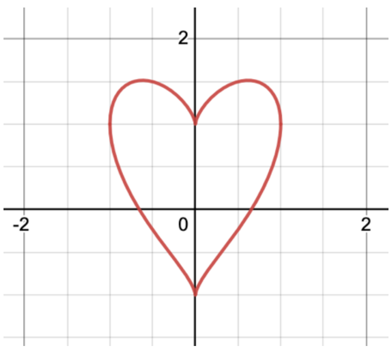
Dear Problem Solvers,

I really hope that you enjoyed and more importantly learned something new by solving problem 8 and problem 9, time to move forward so below are the next two problems.

Problem 10

Proposed by Ivan Retamoso, BMCC, USA.

The Graph of the equation $\left(y - x^{\frac{2}{3}}\right)^2 = 1 - x^2$ is a "heart" and is shown below:



Find the slope of the tangent line of the "heart" at the point $\left(\frac{1}{8}, \frac{2+3\sqrt{7}}{8}\right)$ in exact form.

Problem 11

Proposed by Ivan Retamoso, BMCC, USA.

Find the coordinates of the point (x, y) that belongs to the graph of $x^2 + 18xy + 81y^2 = 144$ that is closest to the origin $(0,0)$ and lies on the third quadrant.