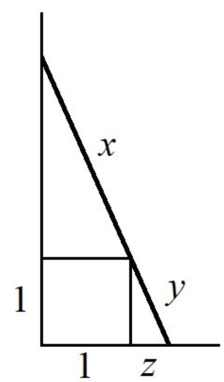
Solution II by Henry Ricardo, Westchester Area Math Circle, Purchase, NY. This problem belongs to a time-honored class of puzzles, often posed in terms of grazing goats or other animals. In his note "The Bull and the Silo: An Application of Curvature" [*Amer. Math. Monthly* **105**(1998), 55-58], Michael E. Hoffman refers to the known answer if the chain has length L and the structure to which the animal is tethered has a circular cross section of radius R: $A = \frac{\pi L^2}{2} + \frac{L^3}{3R}$ if $L \le R\pi$. In our problem, $L = 50\pi$, $R = 50$, so the total area is $A = \frac{\pi(50\pi)^2}{2} + \frac{(50\pi)^2}{3(50)} = \frac{5}{6}(50)^2\pi^3 = \frac{6250\pi}{3}$ square feet.

Two incorrect solutions were also received.

Ladder Leanings

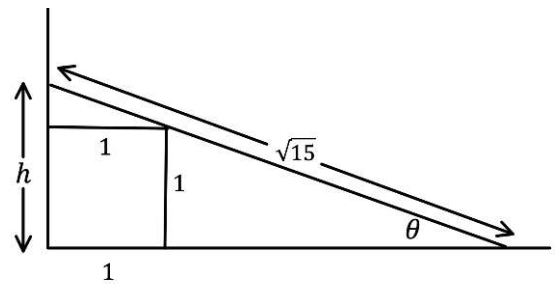
I-2 Borrowed by Michael W. Ecker. A unit cube is placed adjacently to a wall that is at a right angle to the ground. An idealized ladder with length $\sqrt{15}$ leans against the cube and the wall. How high does the ladder reach to the top of the wall?

Solution I by Raymond N. Greenwell (Emeritus), Hofstra University, Hempstead, NY. Let x, y, and z be as in the drawing, where $x + y = \sqrt{15}$ is the length of the ladder.



Notice by similar triangles that $\frac{z}{y} = \frac{1}{x}$, so now $z = \frac{y}{x}$. By the Pythagorean Theorem $1 + z^2 = y^2$, implying $1 + \frac{y^2}{x^2} = y^2$, and then $x^2 + y^2 = x^2y^2$, from which $x^2 + \left(\sqrt{15} - x\right)^2 = x^2\left(\sqrt{15} - x\right)^2$. Then multiplying out and simplifying yields $2x^2 - 2\sqrt{15}\,x + 15 = x^4 - 2\sqrt{15}\,x^3 + 15x^2$. As a result, $x^4 - 2\sqrt{15}\,x^3 + 13x^2 + 2\sqrt{15}\,x - 15 = 0$. We factor to get $(x^2 - \sqrt{15}\,x + 3)(x^2 - \sqrt{15}\,x - 5) = 0$, and using the quadratic formula twice we find four roots, only two of which lie between 1 and $\sqrt{15}$: $\frac{\sqrt{15}+\sqrt{3}}{2}$ and $\frac{\sqrt{15}-\sqrt{3}}{2}$. Finally, the Pythagorean Theorem tells us that the point where the ladder hits the wall is at $1 + \sqrt{x^2 - 1}$. Then substituting and simplifying gives possible heights of $\frac{5+\sqrt{5}}{2} \approx 3.62$ and $\frac{5-\sqrt{5}}{2} \approx 1.38$.

Solution II by Ivan Retamoso. Let h be how high the ladder reaches up the wall, and let θ be the angle formed by the ladder and the ground as shown in the figure below. The triangle on the top of the cube is similar to the triangle on the right of the cube.



Then

$$\frac{\sqrt{15}\sin\theta - 1}{1} = \frac{1}{\sqrt{15}\cos\theta - 1},$$

so $15\sin\theta\cos\theta = \sqrt{15}\sin\theta + \sqrt{15}\cos\theta$. Notice that $0 < \theta < \frac{\pi}{2}$, guaranteeing $\sin\theta$, $\cos\theta$, and $\sin\theta\cos\theta$ are positive, so we divide by the radical and square both sides of the equation to get

$$15\sin^2\theta\cos^2\theta = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta.$$

Simplifying gives $15\sin^2\theta\cos^2\theta - 2\sin\theta\cos\theta - 1 = 0$ or, after factoring, $(3\sin\theta\cos\theta - 1)(5\sin\theta\cos\theta + 1) = 0$. So $\sin\theta\cos\theta = \frac{1}{3}$ but not $\frac{-1}{5}$, since it was guaranteed to be positive. Doubling and using the inverse sine, we get $2\theta = \arcsin\left(\frac{2}{3}\right)$ or $2\theta = \pi - \arcsin\left(\frac{2}{3}\right)$, because $0 < \theta < \frac{\pi}{2}$ says $0 < 2\theta < \pi$. Therefore $\theta = \frac{\arcsin\left(\frac{2}{3}\right)}{2}$ or $\theta = \frac{\pi - \arcsin\left(\frac{2}{3}\right)}{2}$; and since $h = \sqrt{15}\sin\theta$, we obtain $h = \frac{5}{2} - \frac{\sqrt{5}}{2} \approx 1.38$ or $h = \frac{5}{2} + \frac{\sqrt{5}}{2} \approx 3.62$.

Also solved by Austin Jones, Wake Technical Community College, Raleigh, NC; and Troy D. Williamson, Texas State Technical College, Abilene, TX.

m-gon "Scallops"

I-3 Proposed by Stephen L. Plett. Let $m > 2$ be an integer. In a circle of radius m, inscribe a regular m-gon. a) Find S_m,