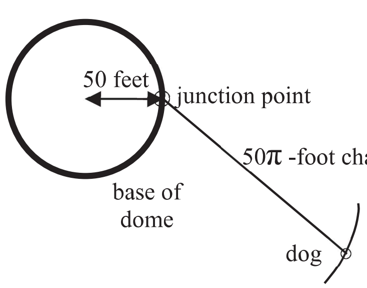
QL-6 **Solution I by the Problems Editor**: First, we note that $\{1,0\}$, $\{2,1\}$, and $\{3,2\}$ are solutions. For example, $3^2 - 2^3 = 1$. Here is why these are the *only* solution pairs in nonnegative integers. For all *real* $x \geq 0$, consider the difference function $d(x) = x^{x+1} - (x+1)^x$. Then we have $d(0) = d(1) = d(2) = -1$, but we also note that $d(3) = 3^4 - 4^3 = 17$. We will show that $d(x)$ never again equals 1 or -1 for any *integer* $x > 3$. (Actually, my proof will even show this is true for all *real* $x \geq 4$, . but we don't need that.) Setting $d(x) = \pm 1$ implies $x^{x+1} = (x+1)^x \pm 1$ or $x \cdot x^x = (x+1)^x \pm 1$. Divide each side by x^x to get $x = \frac{(x+1)^x \pm 1}{x^x} = \left(\frac{x+1}{x}\right)^x \pm \frac{1}{x^x} = \left(1 + \frac{1}{x}\right)^x \pm \frac{1}{x^x}$. That is, (*) $x = \left(1 + \frac{1}{x}\right)^x \pm \frac{1}{x^x}$. Now, for *integral* values of $x > 3$, consider the left and right members of equation (*). For x = 4, the left is 4 and the right is under 2.5; for x = 5, the left is 5 and the right is also under 2.5. As x increases without bound, the left side increases without bound while the right approaches Euler's number, e, about 2.71828. (To be more specific, the first term approaches e while the second one approaches 0.) By the foregoing, there are no integer values of $x \geq 3$ satisfying (*). (Added Note: Marti, the proposer, was very pleased with his discovery, as even the x-values of the solutions, namely 0, 1, 2, are themselves consecutive as well!)

Solution II idea by Steve Plett: For $n \geq 3$, inductively get $n^{(n+1)} - (n+1)^n > \frac{1}{4}(n+1)^n > \frac{1}{4}(4)^3$.

Solutions to Problems from Previous Issues

Walk the Dog

I-1 Proposed by Randy K. Schwartz (retired), Schoolcraft College, Livonia, MI. A guard dog is chained outside a storage building in the shape of a dome, resting on a circular foundation 50 feet in radius.



One end of the chain is attached to the dog's collar, and the other end to a fixed point at the base of the dome. If the chain's length is 50π feet, over how much total area (in square feet) can the dog roam, not counting the area inside the building?

Solution I by Ivan Retamoso, Borough of Manhattan Community College, New York, NY; and (independently) the proposer. Since the chain has length 50π, which is half the circumference of the base of the dome, the chain can be wrapped at most halfway around the base, with the total region swept out by the dog shown as the shaded region in the first figure below (Note that the problem's figure is rotated clockwise by $\frac{\pi}{2}$.). The total area A that can be swept out is the sum of the area A_{lower} below the x-axis and the area A_{upper} above the x-axis in the shaded region. Since A_{lower} is half the area of a circle of radius $R = 50\pi$, then $A_{\text{lower}} = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi(50\pi)^2 = 1250\pi^3$. By symmetry, to compute A_{upper} we need only double the area of the shaded region in the first quadrant. If the chain is pulled tight against the base of the dome, then by the arc length the part of the chain along the base has length 50θ, where the angle θ ranges from 0 to π. Note that the remaining length of the chain is $50\pi - 50\theta = 50(\pi - \theta)$ feet. The area in the first quadrant can now be obtained by summing up infinitesimal sector areas, as shown in the second figure. These sectors have radius $50(\pi - \theta)$ and infinitesimal angle $d\theta$. Using trigonometry, we know that the sectors have area $\frac{1}{2}(\text{radius})^2 \cdot (\text{angle}) = \frac{1}{2}[50(\pi-\theta)]^2 d\theta = 1250(\pi-\theta)^2 d\theta$.

Thus, we have $A_{\text{upper}} = 2\int_0^{\pi} 1250(\pi-\theta)^2 d\theta = -\frac{2500}{3}(\pi-\theta)^3\Big|_0^{\pi} = \frac{2500}{3}\pi^3$. Therefore, the total area is $A = A_{\text{lower}} + A_{\text{upper}} = 1250\pi^3 + \frac{2500}{3}\pi^3 = \frac{6250}{3}\pi^3$ sq. ft.