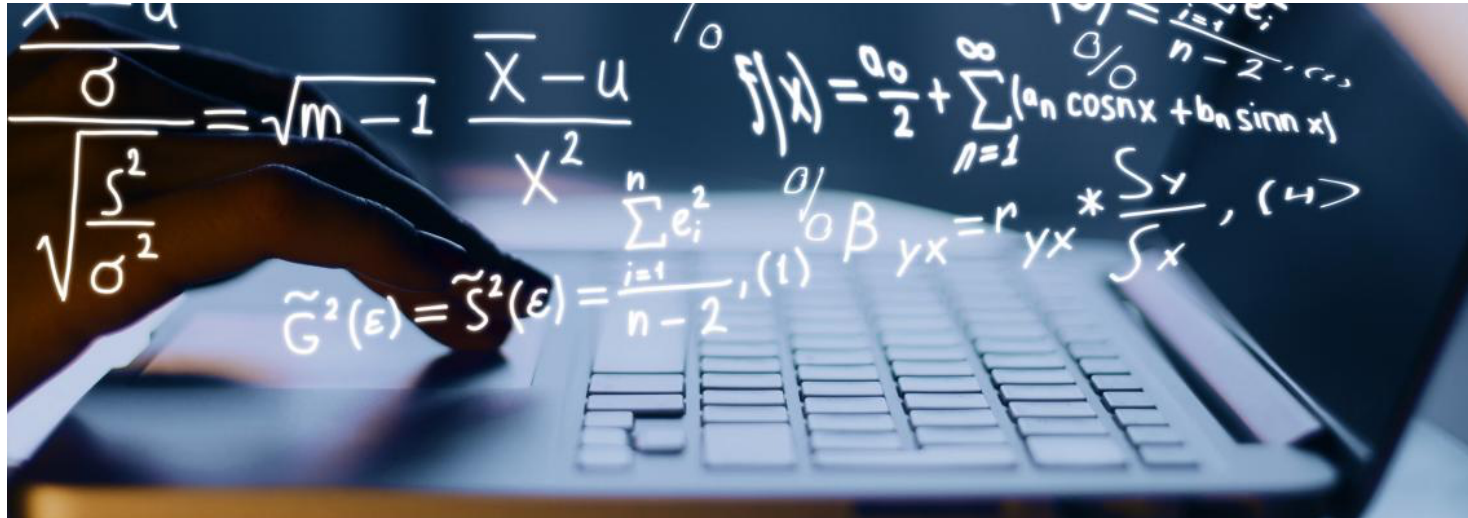


# *Teaching Mathematics using OER, ZTC, and Technology*



nder CC BY-SA

***Now we can!***

***By Ivan Retamoso Ph.D. BMCC***



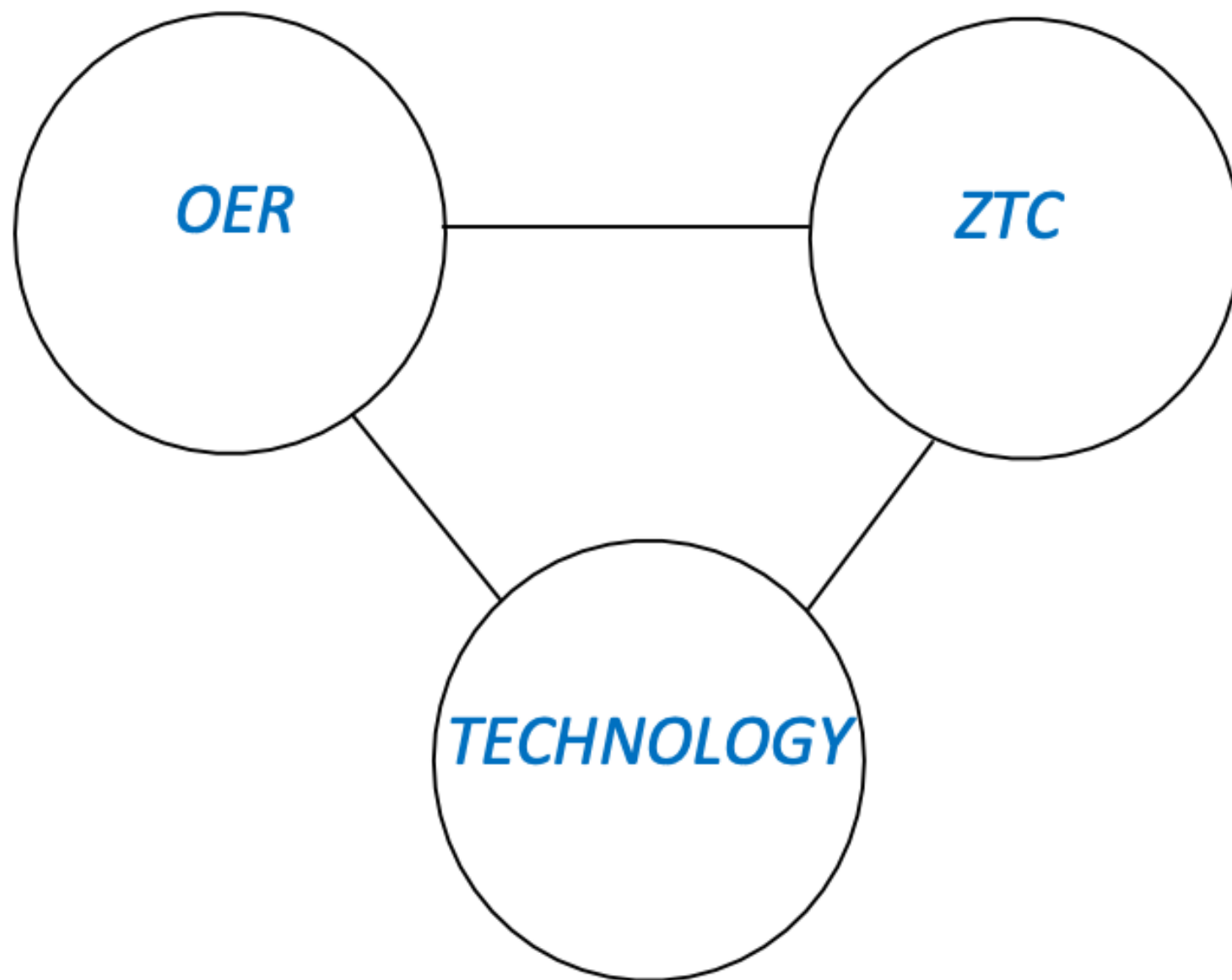




bunch of complicated numbers  
an analytical and structured way to discover the magic of numbers  
massive complex situational difficult everywhere  
part of everything time consuming  
colossal magical more than a subject  
essential intricate conceptual  
daunting  
captivating diverse rewarding  
applicable everything makes sense  
extraordinary an adventure favorite subject  
a tool to better understand the material world  
confusing life savior hard work  
puzzle fascinating  
challenging but perfectly possible  
useful complicated  
logical mathmagic  
fun tedious unique  
life  
logical  
challenging  
logical problem solving

***How can we teach Mathematics such that  
all students are given a fair chance to learn  
regardless of their financial status?***





# *Why do I use OER/ZTC and Technology?*

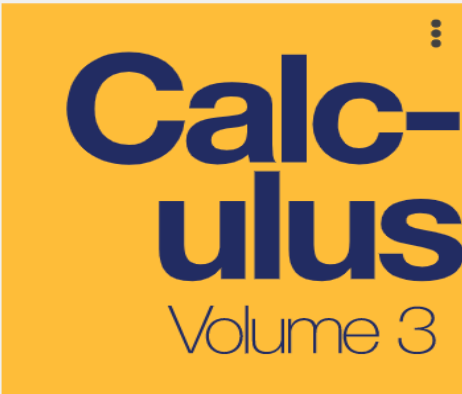
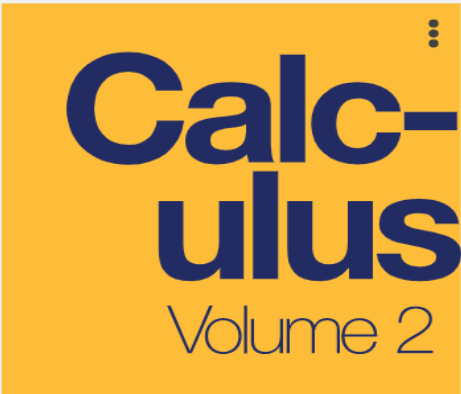
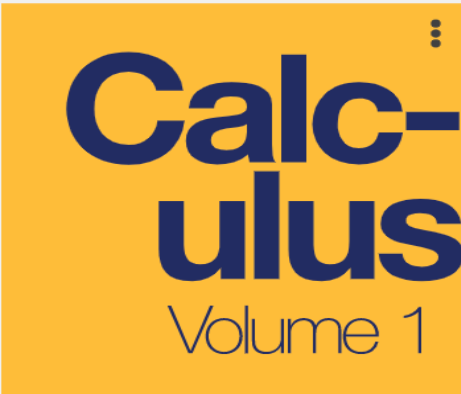
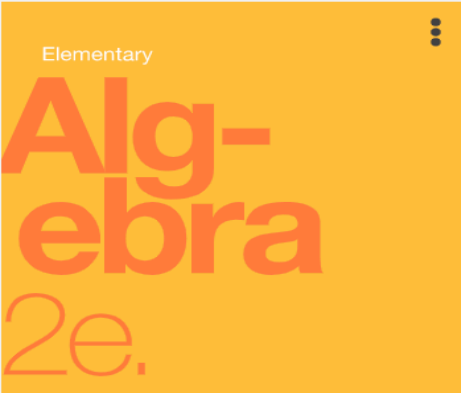
- My students have access to all course materials from day one.
- OER/ZTC are, partly or totally, adaptable and customizable. I can share, remix, transform, and build upon the content.
- As a professor, I have free access to faculty resources, such as Test-Banks, assignments, solutions to homework problems, etc.
- The data is recorded and can be used for future data analysis.
- OER/ZTC lead to increase financial and academic student success.



*Zero Cost TEXTBOOK FROM Openstax*

[www.Openstax.org](http://www.Openstax.org)







# Calculus

Volume 1

Book details

Instructor resources

Student resources

## Academic freedom. No catch.

Access free resources integrated with your book. [Sign up](#) for an account to access locked content.

### Resources for taking your course online

#### Canvas Course Cartridge

The California Community Colleges' Online Education Initiative created a course cartridge for this title to integrate your OpenStax books and resources into the Canvas learning management system. Download your cartridge from the Canvas Commons and work with your campus administrator to add to your course.

 Go

#### Blackboard Course Cartridge

San Jacinto College created a course cartridge for this title to integrate your OpenStax books into the Blackboard learning management system. Download this file and work with your campus Blackboard administrator to add to your course.

 Download

# OPENSTAX TEXTBOOK IMPORTED TO BLACKBOARD



## OpenStax - Calculus Volume 1 Full Textbook



## Preface



## Functions and Graphs



## Limits




## Derivatives



## Applications of Derivatives



## Integration

2020 Fall Term (1)   
Analytic Geometry and  
Calculus I MAT 301  
0516[66450] (Borough of  
Manhattan CC)

Announcements

Course Information

Course Content and  
Assignments (start here)

MAPLE Lab Assignments

Discussion Board

Collaborate

Contact Professor

E-learning Orientation

Help and Resources

Course Dashboard

Tools

CALCULUS I TEXTBOOK

WEBWORK (Homework)

OUR WEBSITE

LECTURE NOTES

RECORDED LECTURES

Free BMCC Online Tutoring

Reviews for Exams

EXAMS

DESMOS Activities

## Derivatives



## Introduction



## Key Terms



## Defining the Derivative



## The Derivative as a Function



## Differentiation Rules



## Derivatives as Rates of Change

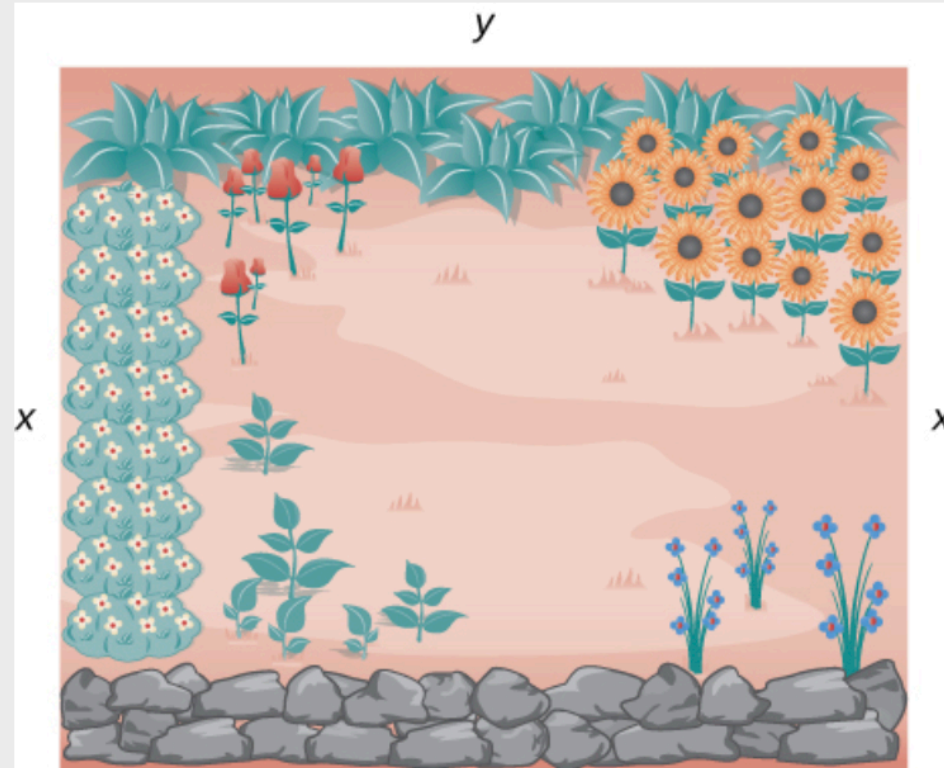


## Derivatives of Trigonometric Functions



**Maximizing the Area of a Garden**

A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides (Figure 4.62). Given 100 ft of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?



**Figure 4.62** We want to determine the measurements  $x$  and  $y$  that will create a garden with a maximum area using 100 ft of fencing.

## 3.6 The Chain Rule

**Rule: The Chain Rule**

Let  $f$  and  $g$  be functions. For all  $x$  in the domain of  $g$  for which  $g$  is differentiable at  $x$  and  $f$  is differentiable at  $g(x)$ , the derivative of the composite function

$$h(x) = (f \circ g)(x) = f(g(x))$$

is given by

$$h'(x) = f'(g(x)) g'(x).$$

3.17

Alternatively, if  $y$  is a function of  $u$ , and  $u$  is a function of  $x$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

**Note:** The Chain Rule

**Rule: Power Rule for Composition of Functions**

For all values of  $x$  for which the derivative is defined, if

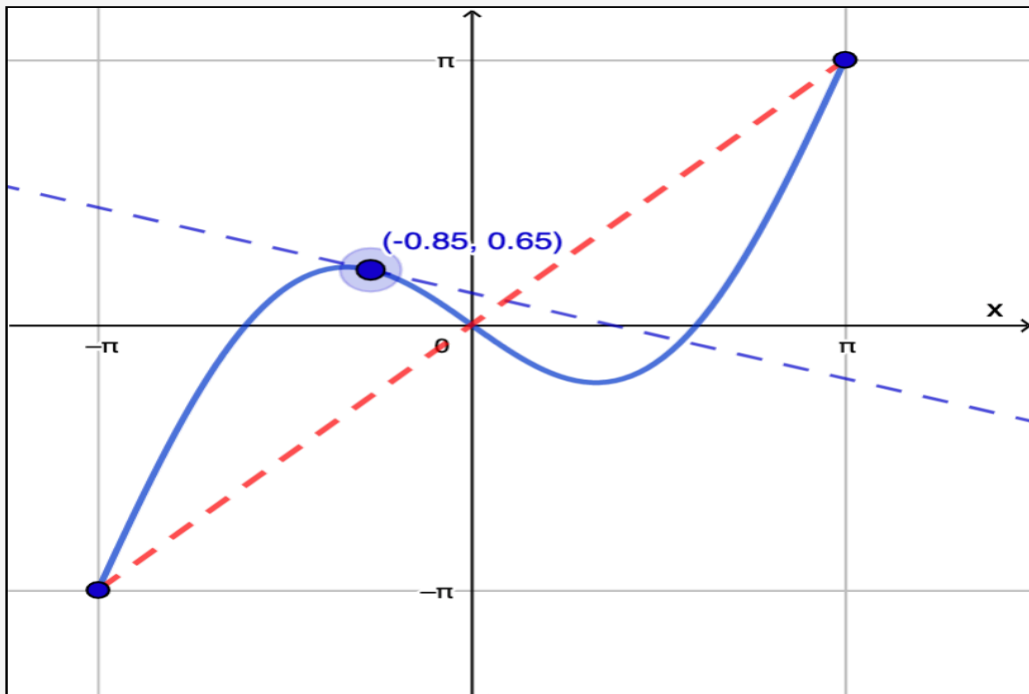
$$h(x) = (g(x))^n.$$

# *Zero cost Homework System: WEBWORK*



[http://webwork.bmcc.cuny.edu/webwork2/2020\\_Fall\\_MAT206\\_0511\\_Retamoso/](http://webwork.bmcc.cuny.edu/webwork2/2020_Fall_MAT206_0511_Retamoso/)

[http://webwork.bmcc.cuny.edu/webwork2/2020\\_Fall\\_MAT301\\_0516\\_Retamoso/](http://webwork.bmcc.cuny.edu/webwork2/2020_Fall_MAT301_0516_Retamoso/)



return this question to its initial state

## WEBWORK Question using “Applet” Related to The Mean Value Theorem

Consider the applet above.

The blue graph shows the function  $f$  that is continuous on  $[-\pi, \pi]$  and differentiable on  $(-\pi, \pi)$ .

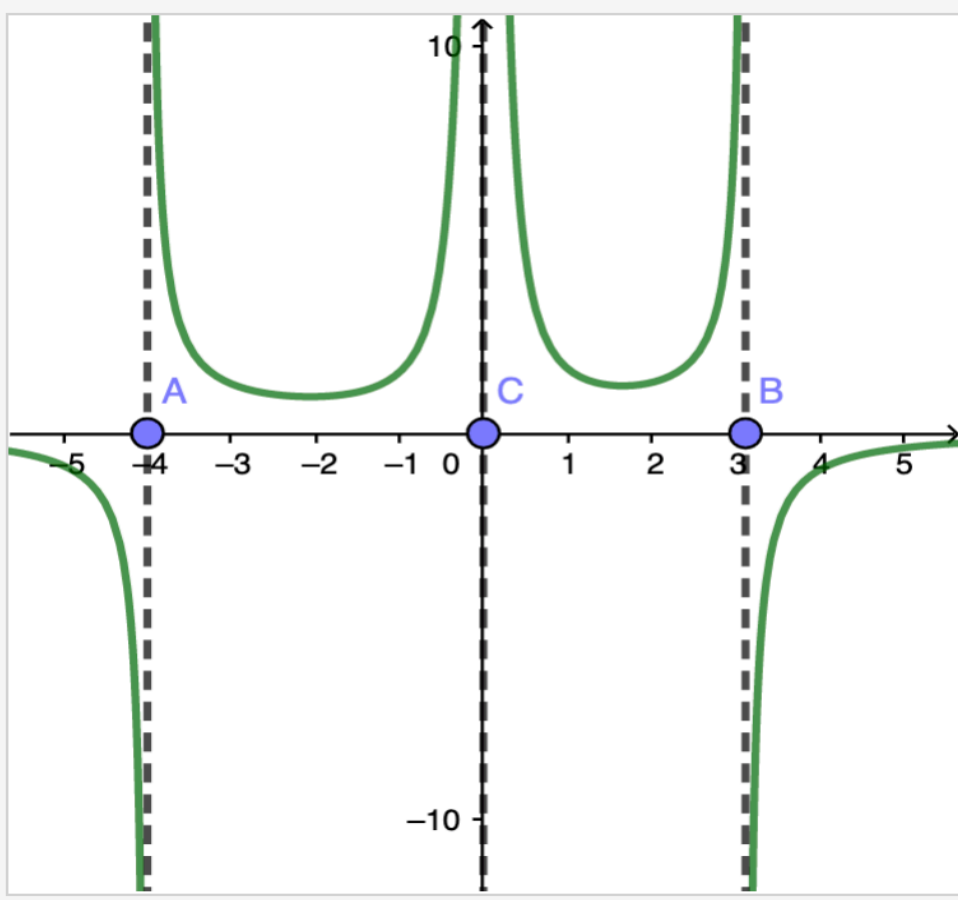
Let  $a = -\pi$  and  $b = \pi$ .

Move the point along the graph to find all the numbers  $c$  between  $a$  and  $b$  so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

List your answers:



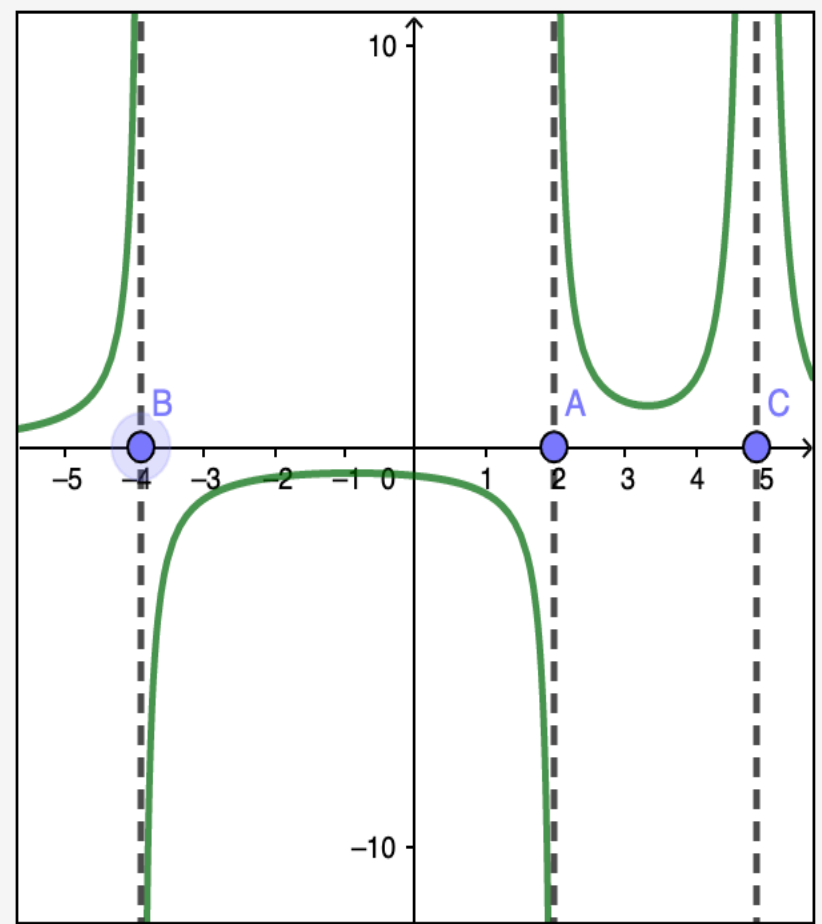


return this question to its initial state

Move the points  $A$ ,  $B$  and  $C$  to construct the graph of the function below.

$$f(x) = \frac{1}{x-2} - \frac{1}{x+4} + \frac{1}{(x-5)^2}$$

Click the submit answers button when you are done.

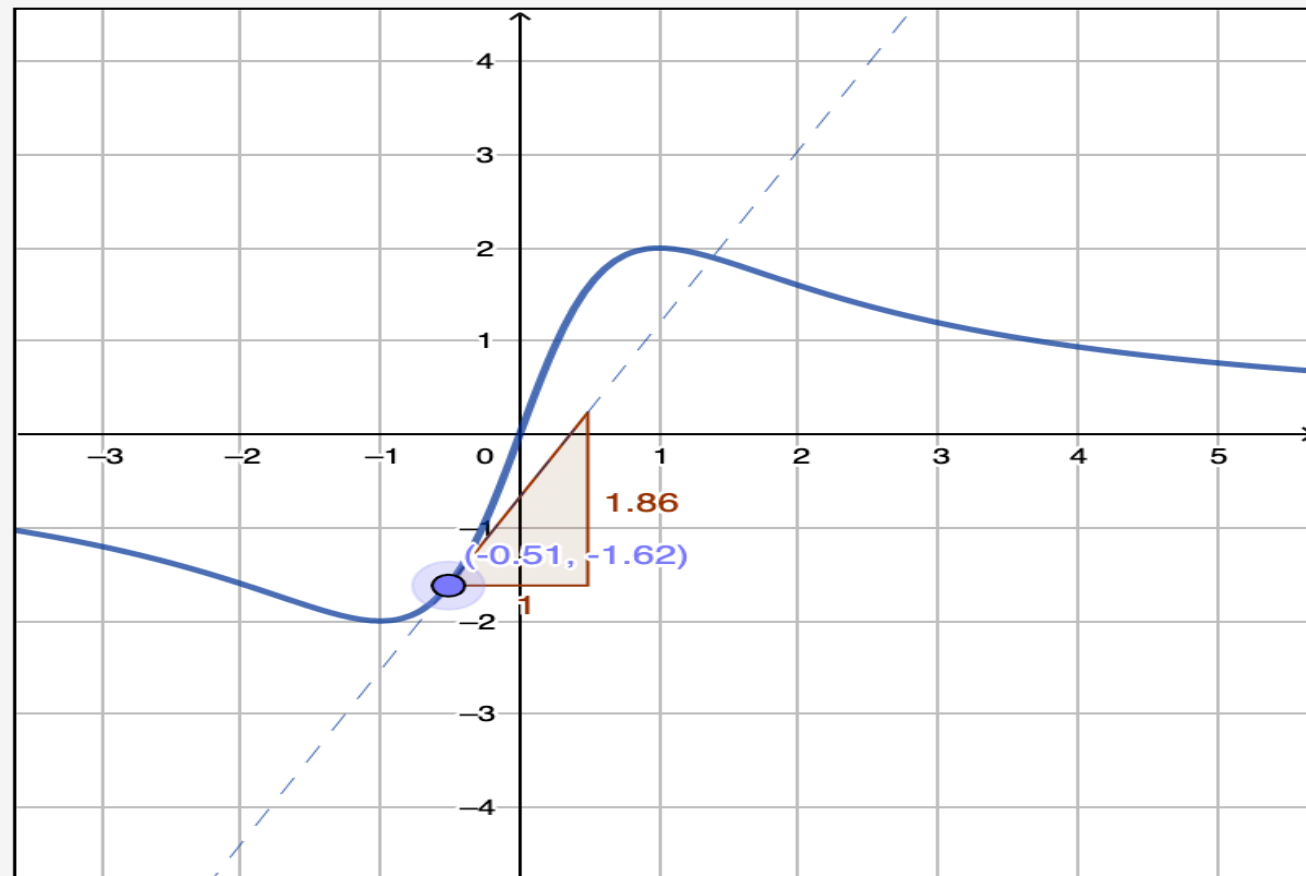


return this question to its initial state

Move the points  $A$ ,  $B$  and  $C$  to construct the graph of the function below.

$$f(x) = \frac{1}{x-2} - \frac{1}{x+4} + \frac{1}{(x-5)^2}$$

Click the submit answers button when you are done. run SetC



return this question to its initial state

Consider the applet above.

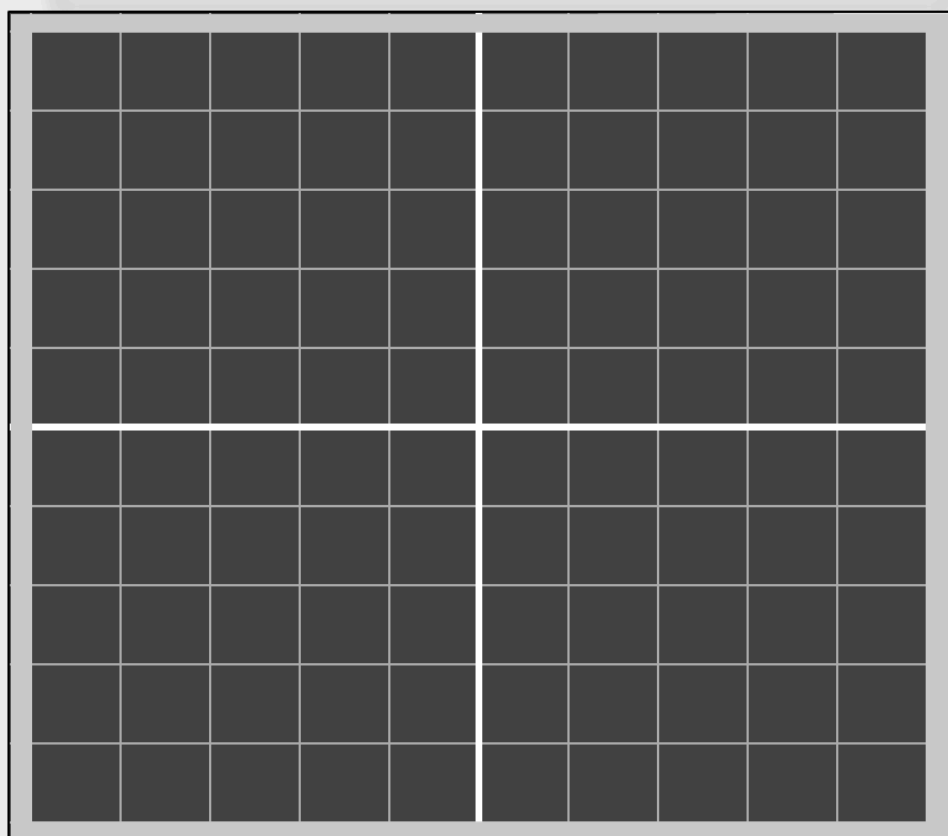
The graph of  $y = f(x)$  is in blue.

Move the point along the graph to compute the slope of the graph.

List all the points-of-inflection.

*WEBWORK Question  
using "Applet"  
Related to Inflection Points*

Draw from left to right across the board. Sketch a positive function with a negative derivative.



ERASE

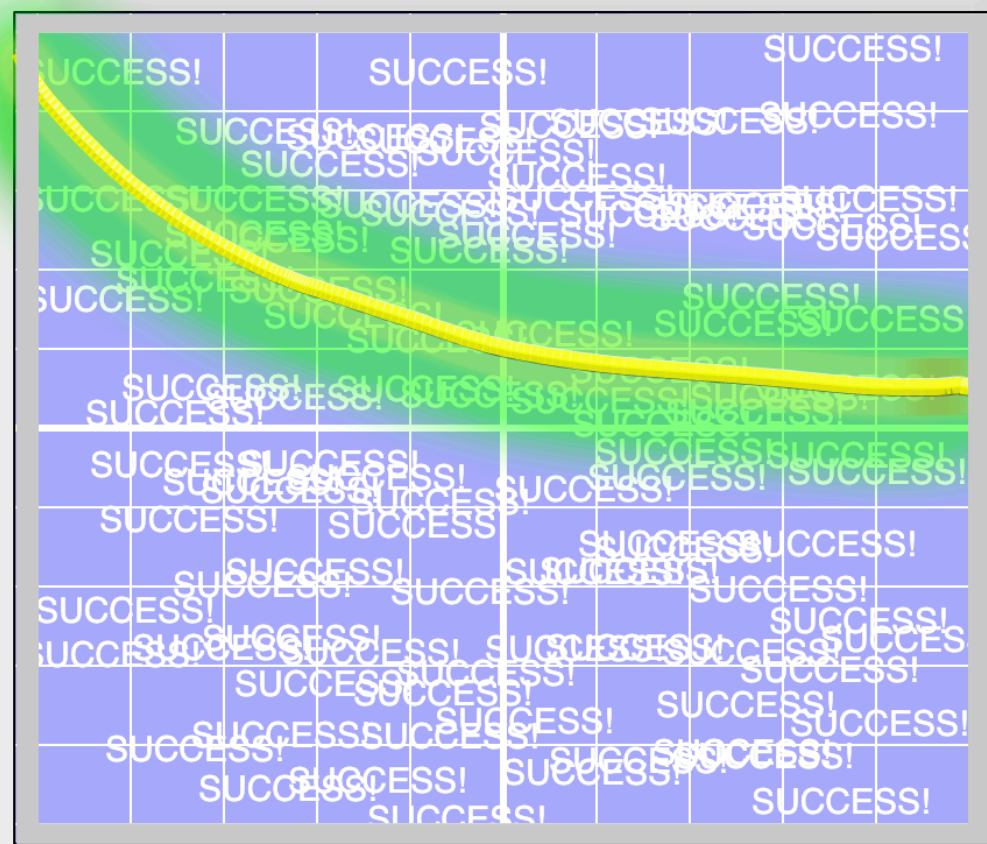
SCORE

SMOOTH

This applet makes use of graphing classes developed by B. Kaskosz and D. Ensley, published at flashandmath.com. This work was supported in part by the National Science Foundation under grant DUE-0941388.

Sketch a positive function with a negative derivative.  
Press 'score' to check your work. When it is correct, press 'submit answers'.  
If your graph is close to correct (not much marked red) pressing the 'SMOOTH' button

Place the cursor over an item to see context sensitive instructions.



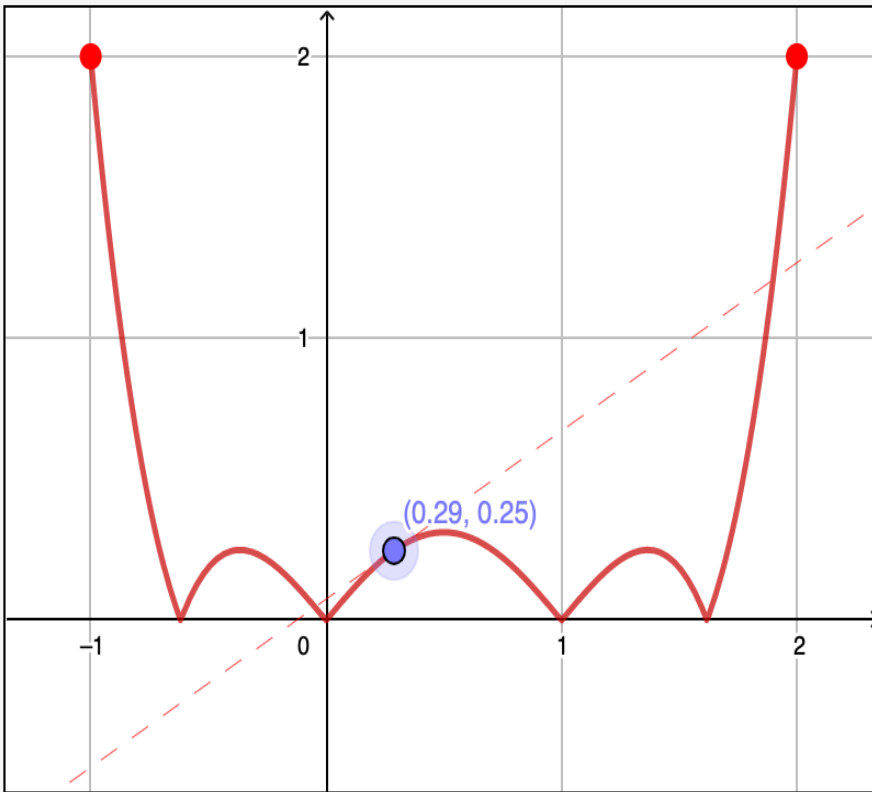
ERASE

SCORE

SMOOTH

This applet makes use of graphing classes developed by B. Kaskosz and D. Ensley, published at flashandmath.com. This work was supported in part by the National Science Foundation under grant DUE-0941388.

Sketch a positive function with a negative derivative.  
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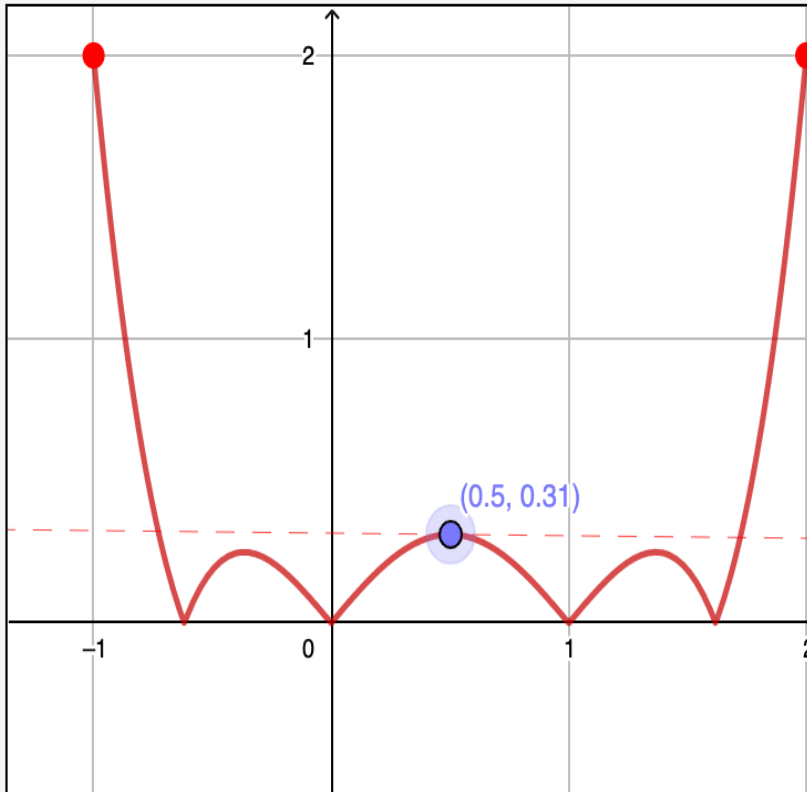
return this question to its initial state

Consider the applet above.

The graph shown is a continuous function on the interval  $[-1, 2]$ .

Move the point along the graph to find all the critical numbers.

List all the critical numbers:



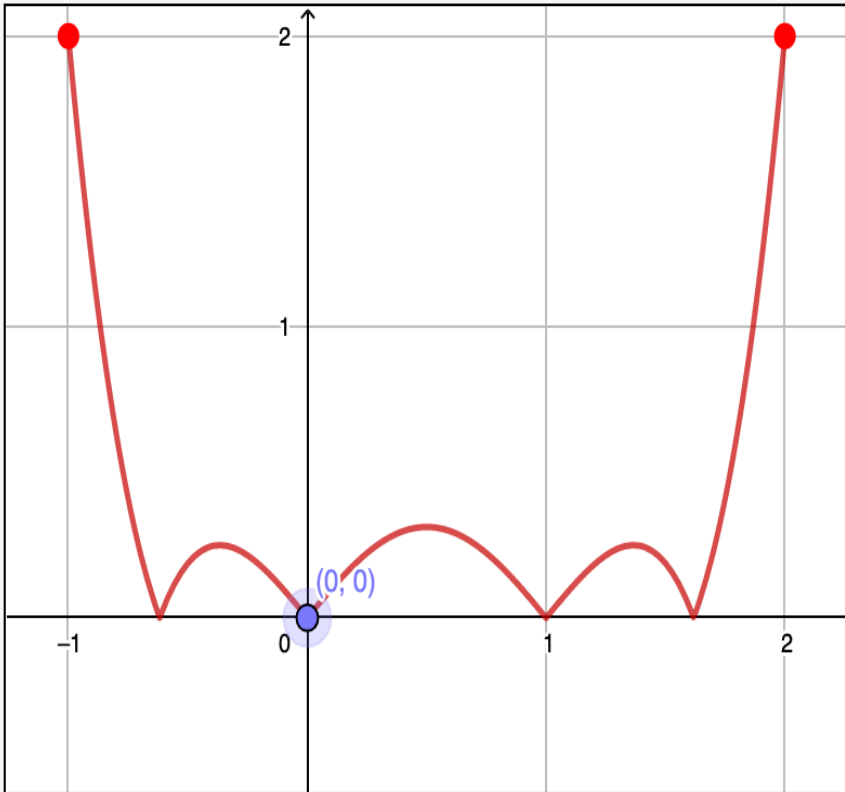
return this question to its initial state

Consider the applet above.

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return this question to its initial state

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Move the point along the graph to find all the critical numbers.

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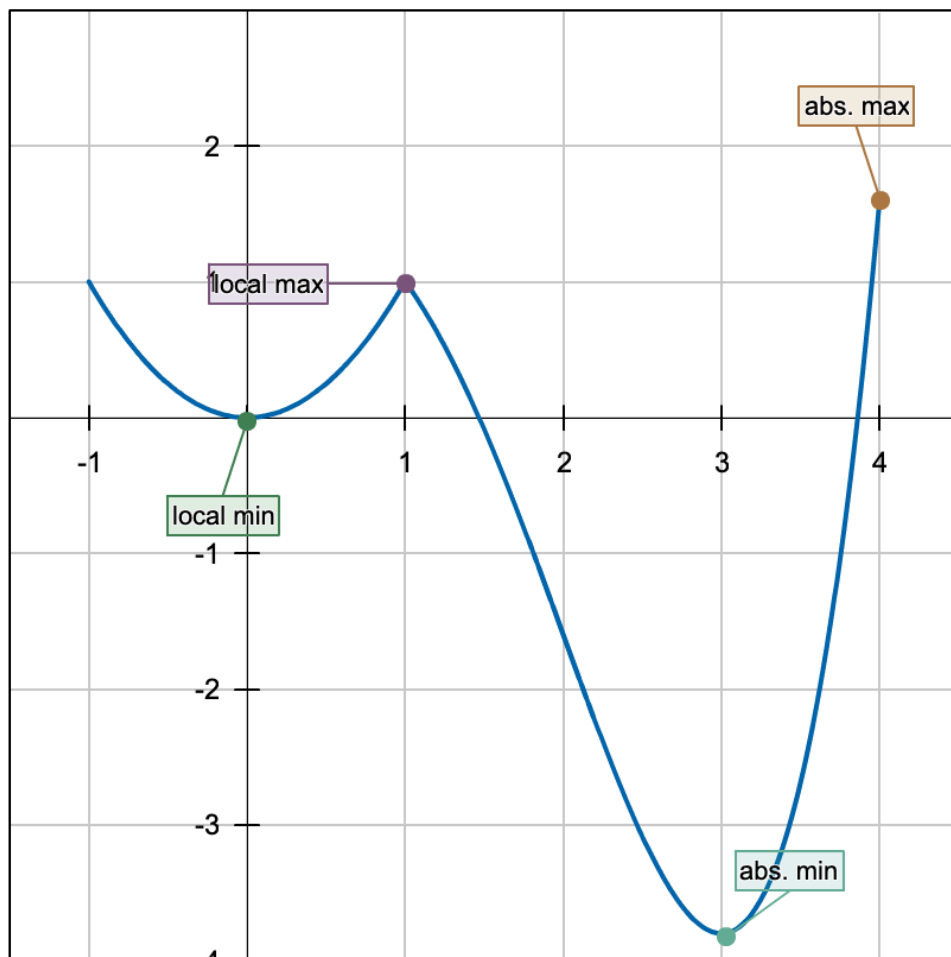


# 4.3 Increasing and Decreasing and the First Derivative Test: Problem 18

[Previous Problem](#)[Problem List](#)[Next Problem](#)

This set is **visible to students**.

(1 point) `local/Label/appletDemonstrationProblems/LabelingPointsCustom.pg`



drag point



delete point

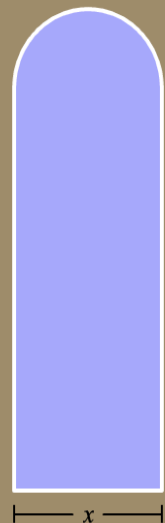
label critical points

label local minima

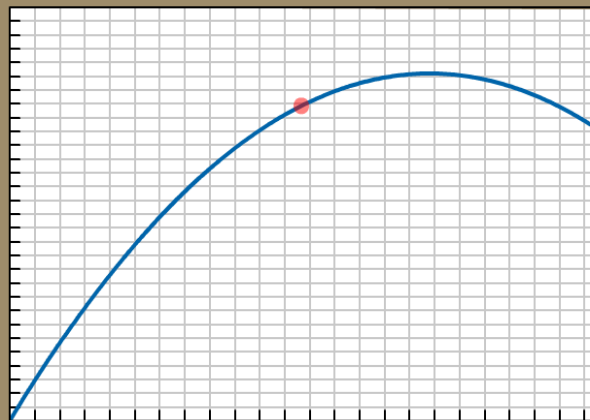
label local maxima

label absolute maximum

label absolute minimum



A Norman window has the shape of a rectangle surmounted by a semicircle. Suppose the outer perimeter of such a window must be 600 cm. Use the slider below to adjust the base length  $x$ . The plot below shows how the area depends on this value.



$x = 116.70$

$A = 22852.00$

## Interactive optimization problem

A Norman window has the shape of a rectangle surmounted by a semicircle. Suppose the outer perimeter of such a window must be 600 cm. To maximize the area of such a window. The applet above shows a plot of the area function. Use the slider to visualize how the area changes for different values of  $x$  and estimate the optimal radius. Then use calculus to find an exact answer. (Correction: In the figure "r" should be "x").

When the base length is zero, the area of the window will be zero. There is also a limit on how large  $x$  can be: when  $x$  is large enough, the rectangle part of the window has zero height. What is the **exact** largest value of  $x$  when this occurs?

largest  $x$ :  cm.

Determine a function  $A(x)$  which gives the area of the window in terms of the parameter  $x$  (this is the function plotted above):

$A(x) =$    $\text{cm}^2$ .

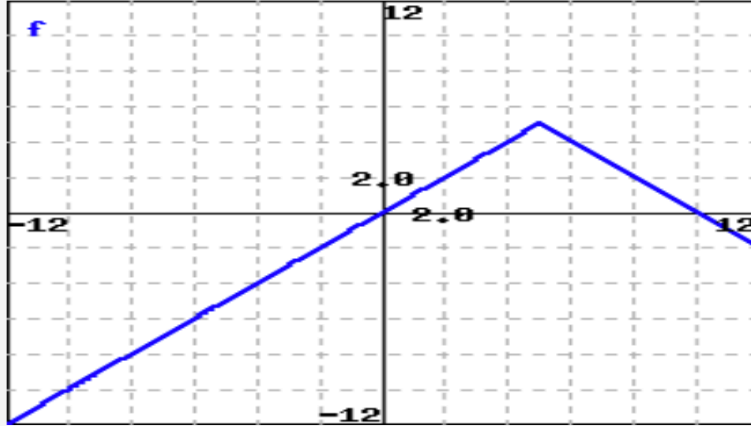
Now find the **exact** base length  $x$  which maximizes this area:

$x =$   cm.

# Embedding your own video in WEBWORK

[Previous Problem](#)[Problem List](#)[Next Problem](#)

(1 point) local/Library/NAU/setFunctionBasicGraphs/Abs.pg



Find the formula for  $f$ .

$f(x) =$

Find the equation of a given graph.

Write an equation for the function graphed below

Step 1:  $f(x) = |x|$

Step 2:  $f(x) = -|x|$

Step 3:  $f(x) = |x| + 3$

Step 4:  $f(x) = |x - 1| + 3$

If the video does not work, [click here to go to YouTube directly.](#)

# Embedding your own video in WEBWORK

[Previous Problem](#)[Problem List](#)[Next Problem](#)

(1 point) **localSCF/CALC1/RelatedRates/05.pg**

An airplane is flying at an altitude of 6 miles and passes directly over a radar station.

When the distance between the plane and the radar station is 10 miles, the radar detects that distance is increasing at 476 mph.



What is the speed of the plane?

Answer:   $\frac{\text{miles}}{\text{hour}}$

Related Rates

When the distance between the plane and the radar station is 10 miles, the radar detects that distance is increasing at 476 mph.

What is the speed of the plane?

At any moment

$$6^2 + x^2 = z^2$$
$$\frac{d}{dt}(36 + x^2) = \frac{d}{dt}(z^2)$$
$$\frac{dx}{dt} = 2z \frac{dz}{dt}$$

When  $z = 10$

$$2(\quad) \frac{dx}{dt} = 2(10)(476)$$

Watch later Share

If the video does not work, [click here to go to YouTube directly.](#)



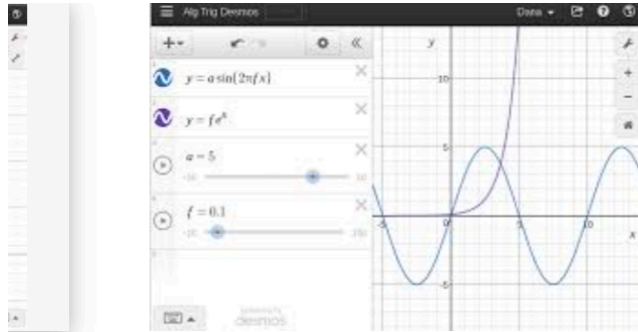
## *Code for embedding video in WEBWORK*

```
$video = MODES(  
HTML=>  
'<iframe width="560" height="315" src="https://www.youtube.com/embed/aeyFb2eVH1c"  
frameborder="0" allow="autoplay; encrypted-media" allowfullscreen></iframe>',  
TeX =>  
"An embedded YouTube video."  
);
```

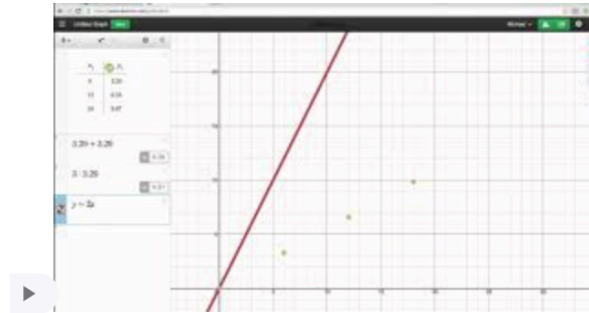
```
BEGIN_TEXT  
${BCENTER}  
$BR
```

```
$video  
$BR  
If the video does not work,  
\{ htmlLink("https://www.youtube.com/embed/aeyFb2eVH1c",  
"click here to go to YouTube directly.") \}  
${ECENTER}  
END_TEXT
```

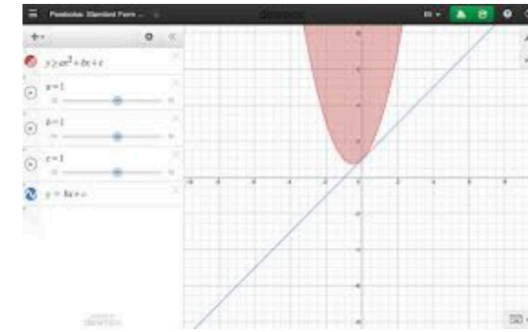
# Free Interactive Activities from DESMOS



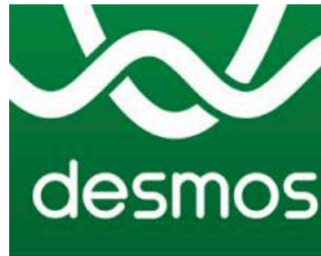
Exponential & Trigonometry notes  
comfsm.fm



Desmos Graphing Calculator - YouTube  
youtube.com

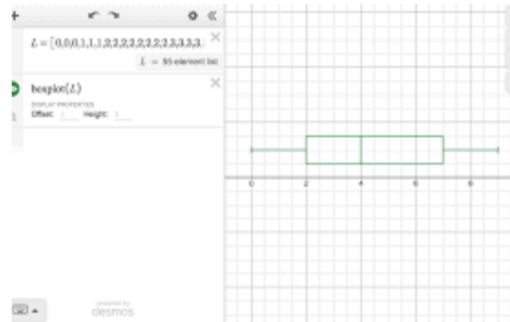


Desmos Graphing Calculator  
chrome.google.com

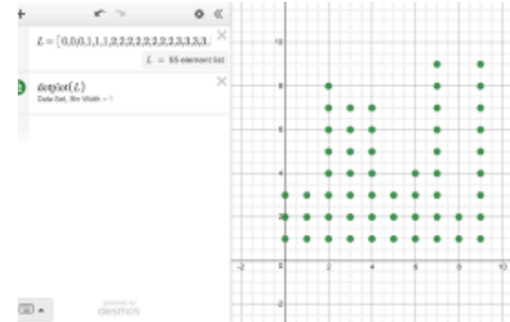


EXPLORE MATH

AASL Recommended Apps: ...  
cmle.org



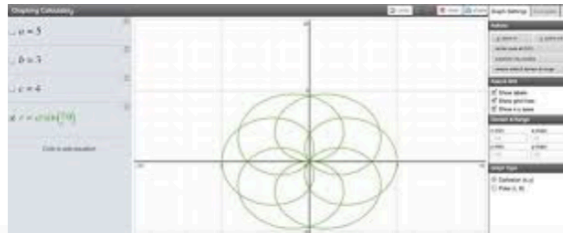
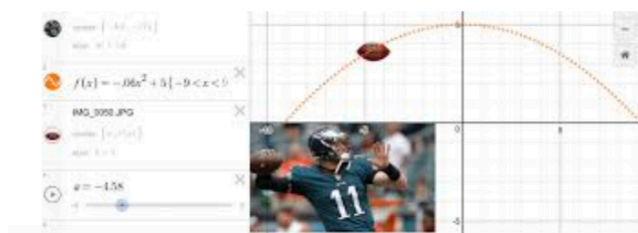
Data Visualizations – Desmos  
support.desmos.com



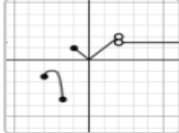
Data Visualizations – Desmos  
support.desmos.com



Introduction to the Desmos Graphing ...  
youtube.com



# Domain and Range Practice.



By Suzanne von Oy **Precalculus**

Mobile Tablet Laptop

Practice with domain and range

Teacher Guide



## Activity Sessions

Assign

SESSIONS		STUDENTS	DATE	
YNQJYP	Expires Feb 15, 2021	34	Aug 19, 2020 at 10:35 pm	<a href="#">View Dashboard</a>
XQRSMR	Expires Nov 21, 2020	20	May 25, 2020 at 8:07 pm	<a href="#">View Dashboard</a>
<a href="#">Inactive Code</a>	<a href="#">Reactivate</a>	16	Feb 5, 2020 at 7:58 am	<a href="#">View Dashboard</a>
<a href="#">Inactive Code</a>	<a href="#">Reactivate</a>	2	Jan 16, 2020 at 9:19 pm	<a href="#">View Dashboard</a>

# DESMOS Activity

## Screens

Student Preview

**1**

A function is graphed in black. Notice that it is a two-piece piecewise function. On the left piece, the orange shaded

**2**

The orange region has now been fixed for the piece to the right, and so in this case, the domain of the function can be

**3**

Practice writing the domain of the function as an inequality in  $f(x)$

**4 Domain Challenge #1**

**5 Domain Challenge #1**

What did you get for the domain of the function on the previous  $f(x)$

**6 Domain Challenge #2**

**7 Domain Challenge #2**

What did you get for the domain of the function on the previous  $f(x)$

**8 Domain Challenge #3**

**9 Domain Challenge #3**

What did you get for the domain of the function on the previous  $f(x)$

**10 Domain Challenge #4**

**11**

Now we will use the same functions, but will identify the range for each. Notice: the range of this function is

**12 Range Challenge #1**

**13 Range Challenge #1**

What did you get for the range of the function on the previous  $f(x)$

**14 Range Challenge #2**

**15 Range Challenge #2**

What did you get for the range of the function on the previous  $f(x)$

**16 Range Challenge #3**

**17 Range Challenge #3**

What did you get for the range of the function on the previous  $f(x)$

**18**

On the next page, there will not be any shaded regions you can play with. Instead, YOU will write the inequalities that match the range, and the shaded regions should show up exactly showing the range of the function.

**19 Range Challenge #4**

**20 Comprehension check #1**

What is the domain and range of this function?

**21 Comprehension check #2**

What is the domain and range of this function?

**22**

Congratulations! You're done!



# Transforming Functions

By Desmos | 30-45 minutes | Practice **Precalculus**

Mobile Tablet Laptop

In this activity, students practice representing graphs of transformations using algebraic notation. The activity gives students timely feedback on their work, letting them see the effect of their algebraic transformations on the graph itself.

Translated by the Desmos localization team into:

French: <https://teacher.desmos.com/activitybuilder/custom/5e8d1a0fc442eb0c0db03794>

Spanish: <https://teacher.desmos.com/activitybuilder/custom/5e8789aed2dcc20bcf9b636b>

Russian: <https://teacher.desmos.com/activitybuilder/custom/5e8fb6d993db0402f4b58d5a>

Teacher Guide



## DESMOS Activity

### Activity Sessions

Assign

SESSIONS	STUDENTS	DATE	
TM4JCM <i>Expires Feb 15, 2021</i>	24	Aug 19, 2020 at 10:39 pm	<a href="#">View Dashboard</a>
Q6S6NG <i>Expires Nov 29, 2020</i>	10	Jun 2, 2020 at 5:32 pm	<a href="#">View Dashboard</a>
Dashboards below are for earlier versions of this activity.			
<i>Inactive Code</i> <a href="#">Reactivate</a>	12	Feb 12, 2020 at 11:57 pm	<a href="#">View Dashboard</a>

### Screens

Student Preview

1 Verbal Transformation #1

Describe in words how you'd transform the black function.

2 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

3 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

4 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

5 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

6 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

7 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

8 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

9 Algebraic Transformati...

Use algebraic notation to describe how to transform the

$f(x)$

10 Algebraic Transforma...

Use algebraic notation to describe how to transform the

$f(x)$

11 Settle a Dispute

Paige and Sai are arguing about this transformation.

12 Card Sort

13 Difficult Transformations

Select which of these transformations is hardest for you to write using algebraic notation.





# Limits and Continuity

By Bryn Humberstone | 30-45 minutes | Introduction **Calculus 1** Distance Learning – ...

Mobile Tablet Laptop

In this activity, students consider left and right limits—as well as function values—in order to develop an informal and introductory understanding of continuity.

Looking for more activities like this?



## Distance Learning – Calculus

**By Desmos** 4 Activities

The activities in this collection are designed for Calculus students, and are especially useful in distance learning settings.

Teacher Guide



# DESMOS Activity

## Activity Sessions

Assign



SESSIONS		STUDENTS	DATE	
37H365	Expires Feb 17, 2021	18	Aug 21, 2020 at 4:10 pm	<a href="#">View Dashboard</a>
5WM9J6	Expires Dec 14, 2020	40	Jun 17, 2020 at 9:39 pm	<a href="#">View Dashboard</a>

## Screens

Student Preview

1 Watch this dot as it m...

What  $y$ -value is the dot getting closer to as  $x$  approaches 1?

$f(x)$

2 Watch this dot as it m...

What  $y$ -value is the dot getting closer to as  $x$  approaches 1?

$f(x)$

3 Now look carefully at t...

What is the actual value of  $y$  when  $x = 1$ ?

$f(x)$

4 Language and Notation

Here's how mathematicians would describe what we saw on

- 
- 
-

5 Continuous?

If all three of these values...

(1) the limit

- 
- 
-

6 Limits

(a) What  $y$ -value is the function approaching as  $x$  approaches 2?

7 Continuous?

Is the function shown here continuous at  $x = 3$ ?

- 
- 
-

8 Make it continuous.

Change the graph (by dragging one of the movable points) to create a function that is continuous at  $x = 3$ .

$x = 3$

9 Sketch a function that ...

(a) What  $y$ -value is it approaching from the left?

10 Sketch a function tha...

As  $x$  approaches 0, the left and right limits equal 2.

- 
- 
-

11 Sketch a function tha...

12 Which of the followin...

- 
- 
-



# Card Sort: Derivative Match

By Desmos

30-45 minutes

Development

Calculus 1



Mobile



Tablet



Laptop



Teacher Guide



## DESMOS Activity

In this activity, students match the graphs of functions with the graphs of their derivatives.

This activity was inspired by a blog post by David Petro:

<http://engaging-math.blogspot.com/2016/06/derivative-matching-cards.html>

### Activity Sessions

Assign



SESSIONS		STUDENTS	DATE	
W6MY23	Expires Feb 17, 2021	8	Aug 21, 2020 at 4:12 pm	<a href="#">View Dashboard</a>
PTFX3Y	Expires Jan 8, 2021	39	Jul 12, 2020 at 11:57 pm	<a href="#">View Dashboard</a>

### Screens

Student Preview

**1** Match each function w...

**2** Consider this function.

Here is one of the cards from earlier.

**3** Consider this derivative.

Here is one of the cards from earlier.

**4** Compare the cards.

Here are two functions from another card sort.

**5** Nice work!

When the activity ends, your teacher will lead a brief class discussion about the key mathematical ideas from this activity.

If you finish early, or if your class is done with the discussion, consider



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<<	Sep 2020			
Mon	Tue	Wed	Thu	Fri
31	<u>1</u> <ul style="list-style-type: none"> <li><u>1.2 Domain and Range</u></li> <li><u>1.3 Rates of Change and Behavior of Graphs</u></li> </ul>	2	<u>3</u> <ul style="list-style-type: none"> <li><u>DESMOS Activity</u></li> <li><u>1</u></li> <li><u>1.4 Composition of Functions</u></li> </ul>	<u>4</u> <ul style="list-style-type: none"> <li><u>Virtual Office Hours</u></li> </ul>
7	<u>8</u> <ul style="list-style-type: none"> <li><u>1.5 Transformation of Functions</u></li> <li><u>1.6 Absolute Value Functions</u></li> </ul>	9	<u>10</u> <ul style="list-style-type: none"> <li><u>1.7 Inverse Functions</u></li> <li><u>2.1 Linear Functions</u></li> </ul>	<u>11</u> <ul style="list-style-type: none"> <li><u>Virtual Office Hours</u></li> </ul>
14	<u>15</u> <ul style="list-style-type: none"> <li><u>DESMOS Activity</u></li> <li><u>2</u></li> <li><u>2.2 Graphs of Linear Functions</u></li> </ul>	16	<u>17</u> <ul style="list-style-type: none"> <li><u>2.3 Modeling with Linear Functions</u></li> </ul>	<u>18</u> <ul style="list-style-type: none"> <li><u>Virtual Office Hours</u></li> </ul>

Calendar  
embedded in  
Website

# Evolution of my Precalculus class

