Verifying Trigonometric Identities (more exercises)

Verify the identity.

1) $\csc^2 x - \cos x \sec x = \cot^2 x$ 1) _____

2) $\dfrac{1 + \csc x}{\sec x} = \cos x + \cot x$ 2) _____

3) $\tan^2 x = \sec^2 x - \sin^2 x - \cos^2 x$

4) $(1 - \cos x)(1 + \cos x) = \sin^2 x$

5) $(\sin x)(\tan x \cos x - \cot x \cos x) = 1 - 2 \cos^2 x$ 5) _____

6) $(\tan x + 1)^2 + (\tan x - 1)^2 = 2 \sec^2 x$ 6) _____

7) $\tan x(\csc x - \sin x) = \cos x$ 7) _____

8) $(1 + \tan^2 x)(1 - \sin^2 x) = 1$ 8) _____

9) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

9) _____

10) $\dfrac{\cos^2 x}{1 - \sin x} = 1 + \sin x$

10) _____

Answer Key
Testname: VERIFYING TRIGONOMETRIC IDENTITIES

1) $\csc^2 x - \cos x \sec x = \csc^2 x - \cos x \cdot \dfrac{1}{\cos x} = \csc^2 x - 1 = \cot^2 x$

2) $\dfrac{1 + \csc x}{\sec x} = \cos x \left(1 + \dfrac{1}{\sin x} \right) = \dfrac{\cos x \, (\sin x + 1)}{\sin x} = \dfrac{\cos x \sin x}{\sin x} + \dfrac{\cos x}{\sin x} = \cos x + \cot x.$

3) $\tan^2 x = \sec^2 x - 1 = \sec^2 x - (\sin^2 x + \cos^2 x) = \sec^2 x - \sin^2 x - \cos^2 x.$

4) $(1 - \cos x)(1 + \cos x) = 1 - \cos^2 x = \sin^2 x$

5) $(\sin x)(\tan x \cos x + \cot x \cos x) = \sin x \left(\dfrac{\sin x \cos x}{\cos x} - \dfrac{\cos^2 x}{\sin x} \right) = \sin^2 x - \cos^2 x = (1 - \cos^2 x) - \cos^2 x = 1 - 2\cos^2$

   x.

6) $(\tan x + 1)^2 + (\tan x - 1)^2 = \tan^2 x + 2 \tan x + 1 + \tan^2 x - 2 \tan x + 1 = 2(\tan^2 x + 1) = 2 \sec^2 x$

7) $\tan x(\csc x - \sin x) = \tan x \cdot \csc x - \tan x \cdot \sin x = \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} - \dfrac{\sin x}{\cos x} \cdot \sin x = \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x} = \dfrac{1 - \sin^2 x}{\cos x} =$

   $\dfrac{\cos^2 x}{\cos x} = \cos x$

8) $(1 + \tan^2 x)(1 - \sin^2 x) = \sec^2 x \cdot \cos^2 x = \dfrac{1}{\cos^2 x} \cdot \cos^2 x = 1$

9) $\sec x + \tan x = \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x} = \dfrac{1 + \sin x}{\cos x} = \dfrac{1 + \sin x}{\cos x} \cdot \dfrac{1 - \sin x}{1 - \sin x} = \dfrac{1 - \sin^2 x}{\cos x(1 - \sin x)} = \dfrac{\cos^2 x}{\cos x(1 - \sin x)} = \dfrac{\cos x}{1 - \sin x}$

10) $1 - \dfrac{\cos^2 x}{1 - \sin x} = 1 - \dfrac{1 - \sin^2 x}{1 - \sin x} = 1 - \dfrac{(1 - \sin x)(1 + \sin x)}{1 - \sin x} = 1 - (1 + \sin x) = -\sin x$